A Moving Horizon State Estimator for Real-Time Stabilization of a Double Inverted Pendulum

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Abstract A moving horizon estimator is designed in the framework of the proximal point minimization algorithm for linear time invariant systems and convergence results are established in the presence of model and measurement noise. The state estimator is implemented in a nonlinear suboptimal feedback control framework for the real time stabilization of a double inverted pendulum on a cart.

1 Introduction

The double inverted pendulum (DIP) is commonly used as a benchmark problem in nonlinear control theory as it is an under-actuated system with highly nonlinear dynamics which is amenable to laboratory study. Control of the DIP can also provide a direct model for systems such as robotic limbs [14], human posture, balance, and gymnast motion [16, 19].

Many control designs have been applied to the DIP system including the linear quadratic regulator (LQR) [7, 9], state dependent Riccati equation control [7], and neural network control [7, 18]. Less work has been done for state estimation of the DIP as many studies use only simulations. State estimation though, is necessary for

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real-time experimental set ups and has been accomplished with Luenburger type observers [11] and low pass derivative filters [6].

The DIP on a cart, consists of two pendula in tandem connected on a hinge to a cart which moves on a linear track as shown in Figure 1. Stabilization control refers to moving the cart along the track, such that the pendula are balanced vertically over the cart in an upright unstable equilibrium. While the pendulum angles and cart position are directly measured, for feedback control of the system, the velocity of the cart and pendulum angles must be estimated.

State estimation for DIP stabilization control has been found to be a challenging task for some methods [5]. The stabilization control of the DIP from an upright start provides an interesting problem, as it can require rapid convergence of state estimates, from a poor initial estimate, in order for the feedback control to rescue the system. The estimation method must also achieve robust performance in the presence of significant model error, measurement error, and disturbances to be effective.

This paper studies state estimation and stabilization control of a DIP on a cart. A nonlinear feedback stabilization control is designed using power series expansion following Garrard [10]. State estimation is approached from a proximal point perspective to construct a moving horizon estimator.

Moving horizon estimation (MHE) most commonly minimizes a least squares cost functional, which includes a regularizing term frequently called the arrival cost, to fit a model trajectory to the N most recent system measurement outputs. Inclusion of an arrival cost term has been found to be important to ensure convergence and stability properties of MHE algorithms [2, 17]. The arrival cost penalizes the difference between the new and previous state estimate and has been interpreted in several ways; as an estimate of the error of the fit to the truncated measurement history when MHE is viewed as approximating the Kalman filter [17], approximating use of an a-priori distribution in probabilistic settings [8], more loosely as a confidence in the past estimate [2], and can also be viewed as a regularization term for solving the state to output inverse problem when MHE is considered as an approximation of a deadbeat observer.

MHE algorithms have been found to converge quickly from poor initial estimates [3,12,17] and to perform robustly in the presence of noise [1,3]. Similar results have been observed in online applications, including charge estimation of batteries [15] and state estimation of a vibrating active cantilever [1].

We approach iterative minimization for state estimation through the framework of the proximal operator and proximal point minimization algorithm. The proximal point minimization algorithm as given in [13] naturally gives rise to a quadratic regulating term similar to the arrival cost terms shown to be effective for MHE in practice. We develop and show convergence of a linear discrete time state estimator in this framework, similar to the MHE given by Alessandri et. al. in [2], and implement it for stabilization of the DIP system.

This paper is organized as follows. In section 2, we construct a proximal point MHE state estimator and give convergence results. In section 3, we introduce the double inverted pendulum system and mathematical model. Section 4 describes a power series based feedback stabilization control, and we implement the control and state estimate on the physical DIP system in section 5.

2 A Proximal Point Moving Horizon Estimator for Linear Time Invariant Systems

Let $\Phi \in \mathbb{R}^{d\times d}$, $W \in \mathbb{R}^{d\times v}$, $C \in \mathbb{R}^{m\times d}$, and controls $u \in \mathbb{R}^{v}$. Consider the following discrete dynamical system

$$\begin{aligned} x_{k+1} &= \Phi x_k + W u_k + \eta_k \\ y_{k+1} &= C x_{k+1} + \varepsilon_k\,, \end{aligned} \tag{1}$$

where $\{\eta_k\}_{k\in\mathbb{N}}$ and $\{\varepsilon_k\}_{k\in\mathbb{N}}$ are unknown model and measurement noise, respectively.

Given the past N system outputs $\{y_{k-i}\}_{i=N-1}^{0}$, an estimate for the state at x_k is constructed with a moving horizon type state estimator using a least squares cost functional, by solving a problem of the form

$$\begin{aligned} &\min_{\{z_{k-j}\}_{j=N-1}^{0}} \;||z_{k-(N-1)} - \hat{x}_{k-(N-1)}||^2 \;+\; \sum_{i=N-1}^{0} ||\, y_{k-i} - C z_{k-i}||^2 \\ &\text{subject to} \quad z_{k-i+1} \;=\; \Phi z_{k-i} \quad \forall i \in 0,1,..N-1\,, \end{aligned} \tag{2}$$

where $\hat{x}_{k-(N-1)}$ is the previous state trajectory estimate and the first term in the cost functional (2) is taken as the arrival cost type term.

We approach the minimization at each iteration of MHE as applying the proximity operator of the trajectory fitting functional to the previous state estimate.

Definition 1. Let the function $\phi : \mathbb{R}^d \to [-\infty,\infty]$ and $\gamma \in]0,\infty[$. The proximity operator of $\gamma\phi$ is defined by

$$Prox_{\gamma\phi} : \mathbb{R}^d \to \mathbb{R}^d : x \to \mathrm{argmin}_{z\in\mathbb{R}^d}\; \phi(z) + \frac{1}{2}\frac{1}{\gamma}||z-x||^2\,.$$

Iterative application of the proximity operator can generate a minimizing sequence for a given cost functional. In particular if ϕ convex, proper, and lower semi-continuous, with argmin $\phi \neq \{\emptyset\}$ then for all $\gamma \in]0,\infty[$, $Prox_{\gamma\phi}$ is well defined. Moreover, for any initial value $z_0 \in \mathbb{R}^d$ with sequence $\{\gamma_k\}_{k\in\mathbb{N}}$ in $]0,\infty[$ such that $\sum_{k\in\mathbb{N}} \gamma_k = \infty$ the proximal point iteration

$$z_{k+1} = Prox_{\gamma_k\phi} z_k$$

generates a minimizing sequence of ϕ [4].

We consider the design of a discrete time state estimator for the system (1) by constructing functionals $\{\phi_k\}_{k\in\mathbb{N}}$ such that $\mathrm{argmin}_{z\in\mathbb{R}^d}\phi_k(z) = x_k$. Then we define the proximal point observer as the sequences $\{\hat{x}_k\}_{k\in\mathbb{N}}$ and $\{p_k\}_{k\in\mathbb{N}}$ constructed according to the recursion

$$\begin{aligned} p_k &= Prox_{\gamma_k \phi_k} \hat{x}_k \\ \hat{x}_{k+1} &= f(p_k)\,. \end{aligned} \tag{3}$$

Functionals capturing least squares type moving horizon estimators can be constructed with a $G \in \mathbb{R}^{(N\cdot m)\times d}$ giving the model to output map and vector $v_k \in \mathbb{R}^{N\cdot m}$ incorporating the model and measurement noise, in the form

$$\phi_k(z) \doteq \frac{1}{2}||G(x_k - z) + v_k||^2\,. \tag{4}$$

For example for the MHE given in (2)

$$G = \begin{bmatrix} C\Phi \\ \dots \\ C\,(\Phi)^N \end{bmatrix}, \qquad v_k = \begin{bmatrix} C\eta_k + \varepsilon_k \\ \dots \\ C\sum_{j=0}^{N-2} \Phi^{N-1-j}\eta_{k+j} + C\eta_{(k+(N-1))} + \varepsilon_{k+N} \end{bmatrix}.$$

Suppose that the matrix $G^T G$ is positive definite, then each ϕ_k has a unique minimizer z_k^*, given by

$$z_k^* = x_k + (G^T G)^{-1} G^T v_k\,.$$

The discrepancy between the minimizer and the true state of the system is denoted as the noise term ζ_k, where for each functional

$$\zeta_k = (G^T G)^{-1} G^T v_k.$$

For convenience the functionals are also adjusted such that the minimum value is zero as follows

$$\phi_k(z) = \frac{1}{2}||G(x_k - z) + v_k||^2 - \frac{1}{2}||(I - G(G^T G)^{-1} G^T)v_k||^2\,,$$

which may be written more conveniently as

$$\phi_k(z) = \frac{1}{2}||G((x_k + \zeta_k) - z)||^2\,. \tag{5}$$

Using functionals of the form (5) the error for the proximal observer estimates (3) follows a simple recursion. For the following it is assumed $G^T G$ is positive definite, with $U \in \mathbb{R}^{d\times d}$ a unitary matrix, and $\Lambda \in \mathbb{R}^{d\times d}$ diagonal such that $G^T G = U\Lambda U^T$. The eigenvalues of $G^T G$ are denoted by $\{\lambda_i\}_{i=1}^d$.

From an initial estimate $\hat{x}_0$, let the sequences $\{\hat{x}_k\}_{k\in\mathbb{N}}$ and $\{p_k\}_{k\in\mathbb{N}}$ be generated according to (3) using the cost functionals (5), with sequence of weighting parameters $\{\gamma_k\}_{k\in\mathbb{N}}$ in $\mathbb{R}_{>0}$.

Proposition 1. *The error terms $e_k = (\hat{x}_k - x_k)$ satisfy the recursion*

$$e_{k+1} = \Phi U \bar{\Lambda}_k U^T e_k \;+\; \Phi U \ddot{\Lambda}_k U^T \zeta_k \;-\; \eta_k$$

Where the diagonal matrices $\bar{\Lambda}_k$, $\ddot{\Lambda}_k$ have entries $\bar{\Lambda}_{i,i} = \frac{1}{1+\gamma_k\lambda_i}$, and $\ddot{\Lambda}_{i,i} = \frac{\gamma_k\lambda_i}{1+\gamma_k\lambda_i}$.

Proof. Here we provide only the main steps of the proof. Note from (3)

$$p_k = Prox_{\gamma_k\phi_{G_k}}(\hat{x}_k) = argmin_{z\in\mathbb{R}^n}\left\{\frac{1}{2}||G(z-(x_k+\zeta_k))||^2 + \frac{1}{2\gamma_k}||z-\hat{x}_k||^2\right\}.$$

Then computing the gradient and setting it equal to zero yields

$$p_k = (G^TG+\frac{1}{\gamma_k}I)^{-1}G^TG(x_k+\zeta_k) \ + \ \frac{1}{\gamma_k}(G^TG+\frac{1}{\gamma_k}I)^{-1}\hat{x}_k\,.$$

Using the fact $G^TG = U\Lambda U^T$,

$$p_k = U(\Lambda+\frac{1}{\gamma_k}I)^{-1}\Lambda U^T x_k \ + \ \frac{1}{\gamma_k}U(\Lambda+\frac{1}{\gamma_k}I)^{-1}U^T\hat{x}_k \ + \ U(\Lambda+\frac{1}{\gamma_k}I)^{-1}\Lambda U^T\zeta_k\,.$$

Therefore,

$$(p_k-x_k) = U((\Lambda+\frac{1}{\gamma_k}I)^{-1}\Lambda - I)U^Tx_k \ + \ \frac{1}{\gamma_k}U(\Lambda+\frac{1}{\gamma_k}I)^{-1}U^T\hat{x}_k \ + \ U(\Lambda+\frac{1}{\gamma_k}I)^{-1}\Lambda U^T\zeta_k\,.$$

Note that

$$(\Lambda+\frac{1}{\gamma_k}I)^{-1}\Lambda - I = -\bar{\Lambda}_k, \qquad \frac{1}{\gamma_k}(\Lambda+\frac{1}{\gamma_k}I)^{-1} = \bar{\Lambda}_k, \qquad (\Lambda+\frac{1}{\gamma_k}I)^{-1}\Lambda = \ddot{\Lambda}_k$$

then

$$(p_k-x_k) = U\bar{\Lambda}_kU^Te_k + U\ddot{\Lambda}_kU^T\zeta_k\,.$$

Therefore,

$$e_{k+1} = (\hat{x}_{k+1}-x_{k+1}) = \Phi(p_k-x_k) - \eta_k = \Phi U\bar{\Lambda}_kU^Te_k + \Phi U\ddot{\Lambda}_kU^T\zeta_k - \eta_k\,.$$

The error recursion of proposition 1, can be used to choose weighting parameters $\{\gamma_k\}_{k\in\mathbb{N}}$ to ensure the error is small relative to the noise. In particular, suppose for all $k\in\mathbb{N}$, $\gamma_k=\gamma$, then the proximal points p_k can be computed more efficiently at each iteration. A criteria for selecting a fixed γ follows immediately from proposition 1.

Proposition 2. *If λ_{min} the smallest eigenvalue of G^TG and $\gamma > max\left\{\frac{||\Phi||-1}{\lambda_{min}}, 0\right\}$, then*

$$\lim_{k\to\infty}||e_k|| \leq \frac{\kappa}{1-r}\,,$$

where $\kappa = ||\Phi||\bar{\zeta}+\bar{\eta}$ and $r = \frac{||\Phi||}{1+\gamma\lambda_{min}}$.

Proof. Using proposition 1 for all $k\in\mathbb{N}$

$$
\begin{aligned}
||e_{k+1}|| &\leq ||\Phi|| \frac{1}{1+\gamma\lambda_{min}} ||e_k|| + ||\Phi|| \frac{\gamma\lambda_{max}}{1+\gamma\lambda_{max}} ||\zeta_k|| + ||\eta_k|| \\
&\leq r||e_k|| + \kappa
\end{aligned}
.
$$

Therefore,

$$
||e_k|| \leq ||e_0|| r^k + \kappa \sum_{m=0}^{k} r^m ,
$$

and $r < 1$, hence

$$
\lim_{k\to\infty} ||e_k|| \leq \frac{\kappa}{1-r} .
$$

2.0.1 Centered Proximal Point Observer

Many cost functionals of the form (5) can be constructed for discrete systems (1). A functional utilizing only the three most recent measurements was found to be effective for the DIP state estimation problem. For all $k \in \mathbb{N}$ let the function $\phi_{cntr_k} : \mathbb{R}^d \to \mathbb{R}$ be defined as

$$
\phi_{cntr_k}(z) \doteq \frac{1}{2}||C(\Phi^{-1}z + W^{-1}u_{k-1}) - y_{k-1}||^2 + \frac{1}{2}||Cz - y_k||^2 + \frac{1}{2}||C(\Phi z + Wu_k) - y_{k+1}||^2 . \quad (6)
$$

Then with $v_k \in \mathbb{R}^{3\cdot m}$ and $G \in \mathbb{R}^{(3\cdot m)\times d}$ given by

$$
G = \begin{bmatrix} C\Phi^{-1} \\ C \\ C\Phi \end{bmatrix} \quad \text{and} \quad v_k = \begin{bmatrix} -C\Phi^{-1}\eta_{k-1} + \varepsilon_{k-1} \\ \varepsilon_k \\ C\eta_k + \varepsilon_{k+1} \end{bmatrix} ,
$$

ϕ_{cntr_k} is of the form (5).

3 Mathematical Model of the Double Inverted Pendulum

The DIP system used in this work was provided by Quanser Consulting Inc. and consists of an upper (12 in.) aluminium rod connected on a hinge to a lower (7 in.) rod which is in turn connected on a hinge to a cart (an IPO2 linear servo unit) that moves on a track as shown in Figure 1. Encoders measure three variables; the position of the cart (x_c), the angle between the lower pendulum and normal vector vertical to the cart (α), and the angle between the lower and upper pendulum (θ). The measurements are defined such that upright unstable equilibrium is the origin and counterclockwise rotation is positive. The system is controlled through voltage input to a DC motor that moves the cart along the track.

The model for the DIP is derived using Lagrange's energy method as is commonly done, for example in, [7, 9, 11]. A full derivation and description of the model can be found in [5,6].

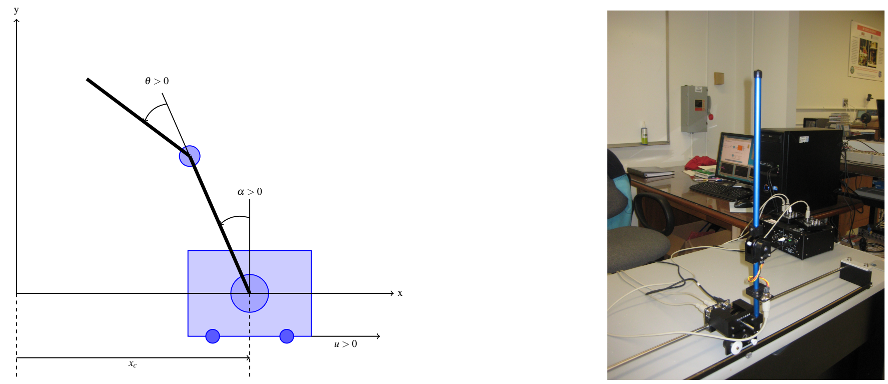

Fig. 1: Diagram and photo of the DIP system.

4 Power Series Stabilization Controller

For an $f : \mathbb{R}^6 \to \mathbb{R}^6$ and $B \in \mathbb{R}^6$, with control, $u : [0,\infty[\to \mathbb{R}$, the DIP dynamics are given by a system of the form

$$\dot{x} = f(x) + Bu \qquad \text{for } \; x = [x_c, \theta, \alpha, \dot{x}_c, \dot{\theta}, \dot{\alpha}]^T \, .$$

A feedback stabilization control is constructed with respect to the cost functional

$$J(x_0, u) = \frac{1}{2} \int_0^\infty x^T Q x + R u^2 dt \, , \tag{7}$$

with $Q \in \mathbb{R}^{6\times 6}$ and $R \in \mathbb{R}$, symmetric positive definite matrices. The optimal feedback control is

$$u(x) \doteq -R^{-1} B^T V_x(x) \, ,$$

where V is the solution to the corresponding Hamilton Jacobi Bellman equation, which is approximated using power series expansions following [10]. Let f be expanded about the unstable equilibrium as

$$f(x) = Ax + \sum_{n=2}^{\infty} f_n(x), \;\; \text{where } f_n(x) = O(|x|^n) \, .$$

Then for P the solution of the algebraic Riccati equation corresponding to the linearized system and cost functional (7), the control for the DIP is given in the feedback form

$$u^*(x) = -R^{-1} B^T \left[Px - \left(A^T - PBR^{-1} B^T \right)^{-1} P f_3(x) \right] . \tag{8}$$

Full details can be found in [5].

5 Real time stabilization of a double inverted pendulum

The centered proximal point observer was applied to the DIP system by first constructing a discrete system of the form (1) using a linearization of the the nonlinear DIP model about the unstable equilibrium. The state transition matrices Φ, Φ^{-1}, W, and W^{-1} for the linearized system were approximated using Matlab's expm command. A fixed $(\gamma > 0)$ was used in the computation of state estimates according to proximal point observer (3) using the cost functionals (6), which at each iteration requires a solution to

$$p_k = \text{argmin}_{z\in\mathbb{R}^6} \frac{1}{2}||Hz - q_k||^2 , \tag{9}$$

for

$$H = \begin{bmatrix} \frac{1}{\gamma} I \\ C\Phi^{-1} \\ C \\ C\Phi \end{bmatrix} \text{ and } q_k = \begin{bmatrix} \frac{1}{\gamma}\hat{x}_k \\ y_{k-1} - CW^{-1}Bu_{n-1} \\ y_k \\ y_{k+1} - CWBu_n \end{bmatrix} .$$

To compute (9), an offline QR factorization for $H = Q_H R_H$ was computed with the Matlab qr command. Then the centered proximal point moving horizon estimation (CPX) with a fixed γ was iterated from initial estimate $\hat{x}_0 = 0$ according to

$$\begin{aligned} p_k &= R_H^{-1} Q_H^T q_k \\ \hat{x}_{k+1} &= \Phi p_k + W u_k . \end{aligned} \tag{10}$$

When implemented for the DIP feedback control in real time, the estimates supplied to compute the control were the model predictions $\tilde{x}_{k+2} = \Phi\hat{x}_{k+1} + u_{k+1}$.

The power series feedback stabilization control was computed with

$$Q = diag([80, 300, 100, 0, 0, 0]) \text{ and } R = .5 ,$$

where the state of the system is $x = [x_c, \theta, \alpha, \dot{x_c}, \dot{\theta}, \dot{\alpha}]^T$.

The estimator and control were implemented in real time through MATLAB Simulink interfaced with Quanser's Quarc software on a desktop computer running Windows 7 with a 3.20 GHz Intel Core i5 650 processor and 4 GB of RAM, connected to the DIP system by two Q2-USB DAQ control boards, with the control voltage applied to the cart by a VoltPAQ amplifier.

For a comparison study, both a CPX and a second order low pass derivative filter (LDF) were used to supply state estimates for stabilization control. The CPX estimator (10) was applied with $\gamma = 150$, while the LDF was used with Quanser's supplied parameters: cutoff frequency $\omega = 100\pi$ for the cart, $\omega = 20\pi$ for the pendulum angles, and damping ratios .9. Stabilization control was initiated once measurement values were brought to within .01 of the balanced state, the average value and variance for the measured DIP states when under stabilization control with CPX and LDF are reported in Table 1. Feedback control using the CPX estimates maintained the system closer to the balanced state and with less variance than with LDF estimates.

	x_c (cm)		α (degrees)		θ (degrees)	
	mean	variance	mean	variance	mean	variance
CPX	-.002	5.52	.178	35.2	-.45	3.89
LDF	-.119	5.98	1.21	41.8	-.867	6.78

Table 1: Output of DIP stabilization over (6.5 sec) interval using either centered proximal point MHE (CPX) or low pass derivative filter (LDF) to compute the feedback control, the stabilized state is the origin

The CPX and LDF differed most for the estimates of the rate of change for θ, the angle between the pendulums. Figure 2 shows a comparison between CPX and LDF angle velocity estimates using measurement data from the physical DIP system under stabilization control.

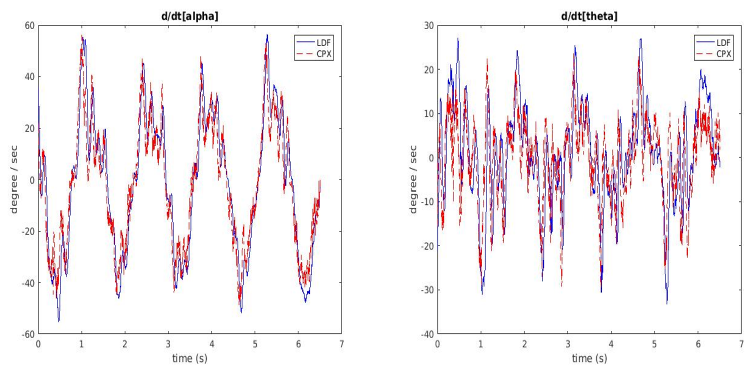

Fig. 2: Comparison of CPX and LDF angle velocity estimates for the real time DIP system under stabilization control.

The criteria for γ to guarantee CPX estimate convergence given in Proposition 2 is $\gamma \geq 24400$ for the DIP system. When values of γ satisfying the condition were used the CPX angle velocity estimates had large amplitude high frequency oscillations unsuitable for computing a control, γ was reduced two orders of magnitude from the Proposition 2 criteria before a reasonable control could be computed. The need for a smaller γ value is likely due to H in (9) becoming more ill conditioned for larger γ and the solutions of (9) more sensitive to noise.

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